Math B1131, Quiz #6, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the area bounded by the curves $2y = 4x - x^2$ and $2y = x - 4$. Sketch the region.

$4x - x^2 = x - 4$

$0 = x^2 - 3x - 4$

$(x-4)(x+1)$

$x = 4, x = -1$

$y = 2x - \frac{1}{2}x^2$ $y = \frac{1}{2}x - 2$

$\int_{-1}^{4} 2x - \frac{1}{2}x^2 - \frac{1}{2}x + 2 \, dx = \int_{-1}^{4} -\frac{1}{2}x^2 + \frac{3}{2}x + 2 \, dx =$

$-\frac{1}{6}x^3 + \frac{3}{4}x^2 + 2x \Big|_{-1}^{4} = -\frac{1}{6}(4)^3 + \frac{3}{4}(4)^2 + 2(4) + \frac{1}{6}(-1)^3 - \frac{3}{4}(-1)^2 - 2(-1) =$

$\frac{-64}{6} + 12 + 8 - \frac{1}{6} - \frac{3}{4} + 2 = \frac{-65}{6} + 22 - \frac{3}{4} = \underline{\frac{125}{12}}$

2. For the demand equation $p = \frac{50}{q+5}$, and the supply equation $p = \frac{q}{10} + 4.5$, find the consumer's and producer's surplus.

$\frac{50}{q+5} = \frac{q}{10} + 4.5 = \frac{1}{10}(q + 45)$

$500 = (q+5)(q+45)$

$500 = q^2 + 50q + 225$
$-500 \qquad\qquad -500$

$0 = q^2 + 50q - 275$

$0 = (q-5)(q+55)$

$q = 5$ $\cancel{q = -55}$

$p = \frac{50}{5+5} = \frac{50}{10} = 5$

$(5,5)$

$CS = \int_0^5 \frac{50}{q+5} - 5 \, dq =$

$50\ln|q+5| - 5q \Big|_0^5 =$

$50\ln 10 - 25 - 50\ln 5 =$

$50 \ln \frac{10}{5} - 25 = \boxed{50\ln 2 - 25}$

≈ 9.66

$PS = \int_0^5 5 - \frac{q}{10} - 4.5 \, dq = \int_0^5 \frac{1}{2} - \frac{q}{10} \, dq$

$\frac{1}{2}q - \frac{q^2}{20} \Big|_0^5 = \frac{5}{2} - \frac{25}{20} - 0 = \frac{5}{4} = \boxed{1.25}$

$(5/4)$

3. Explain why $\int x\, e^x dx$ is integration by parts, and $\int x\, e^{x^2} dx$ is not.

in $\int x e^{x^2}$

if $u = x^2$

$du = 2x\,dx$

that x is needed to integrate e^{x^2}

no such substitution is possible for $\int x e^x dx$

the product functions are not related (by chain rule) and so required by parts.

4. Integrate $\int x^4 \ln x\, dx$ by parts.

$u = \ln x \qquad dv = x^4\,dx$

$du = \frac{1}{x} \qquad v = \frac{1}{5}x^5\,dx$

$\frac{1}{5}x^5 \ln x - \int \frac{1}{5}x^5 \cdot \frac{1}{x}\,dx$

$\frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4\,dx$

$\frac{1}{5}x^5 \ln x - \frac{1}{5}\cdot\frac{1}{5}x^5 + C$

$\boxed{\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C}$