Math B1131, Exam #2, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Given the demand equation $p = 900 - q^2$, and the supply equation $p = 10q+300$, find the Consumers' surplus and Producers' surplus. (20 points)

$900 - q^2 = 10q + 300$

$0 = q^2 + 10q - 600$

$0 = (q-20)(q+30)$

$q_0 = 20,\ \cancel{q = -30}$

$P_0 = 10(20) + 300 = 200 + 300 = 500$

$CS = \int_0^{20} 900 - q^2 - 500\, dq$

$\int_0^{20} 400 - q^2\, dq =$

$400q - \frac{1}{3}q^3 \Big|_0^{20} =$

$400(20) - \frac{1}{3}(20)^3 =$

$8000 - \frac{8000}{3} =$

\$5,333.33

$PS = \int_0^{20} 500 - (10q+300)\, dq$

$\int_0^{20} 500 - 10q - 300\, dq$

$\int_0^{20} 200 - 10q\, dq$

$200q - 5q^2 \Big|_0^{20}$

$200(20) - 5(20)^2 =$

$4000 - 2000 =$

\$2,000

2. Integrate by whatever means appropriate. (15 points each)

a. $\int x^2 e^{3x} dx$ by parts

$uv - \int v\,du = \int u\,dv$

$u = x^2 \quad dv = e^{3x}$

$du = 2x\,dx \quad v = \frac{1}{3}e^{3x}$

$\frac{1}{3}x^2 e^{3x} - \int 2x \cdot \frac{1}{3}e^{3x}\, dx$

$u = 2x \quad dv = \frac{1}{3}e^{3x}$

$du = 2\,dx \quad v = \frac{1}{9}e^{3x}$

$\frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \int \frac{2}{9}e^{3x}\, dx = \boxed{\frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C}$

tabular method

	u	dv
+	x^2	e^{3x}
−	$2x$	$\frac{1}{3}e^{3x}$
+	2	$\frac{1}{9}e^{3x}$
−	0	$\frac{1}{27}e^{3x}$

$= \frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

same

b. $\int \sqrt{x}\ln(x)\,dx$ by parts $\int u\,dv = uv - \int v\,du$

$u = \ln x \qquad dv = x^{1/2}\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{2}{3}x^{3/2}$

$\frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx =$

$\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx$

$\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + C = \boxed{\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C}$

c. $\int \frac{x+10}{x^2-x-12}dx = \int \frac{x+10}{(x-4)(x+3)}dx = \int \frac{A}{(x-4)} + \frac{B}{(x+3)}dx =$

$A(x+3) + B(x-4) = x+10$

$x=-3$

$A(0) + B(-3-4) = -3+10$

$-7B = 7$

$B = -1$

$x = 4$

$A(7) + B(0) = 4+10$

$7A = 14$

$A = 2$

$\int \frac{2}{x-4}dx + \int \frac{-1}{x+3}dx =$

$\boxed{2\ln|x-4| - \ln|x+3| + C}$

3. Use the rules for partial fractions to set up the integral $\int \frac{4x^3-3x^2+2x-3}{(x^2+3)(x-2)^2(x+1)}dx$ with appropriate simpler terms and constants. Do not attempt to find the constants or to integrate the result. (10 points)

$\boxed{\frac{Ax+B}{x^2+3} + \frac{C}{(x-2)} + \frac{D}{(x-2)^2} + \frac{E}{(x+1)}}$

4. Suppose that the population of a certain town in 1960 was 50,000, and it doubled over the next 50 years. How big will the population be in 2025? (15 points)

$k = \frac{\ln 2}{50}$

$2025 - 1960 = 65$

$P = 50{,}000\, e^{\frac{\ln 2}{50} t}$

$P(65) = 50{,}000\, e^{\frac{\ln 2}{50}(65)} = \boxed{123{,}114 \text{ people}}$

5. Solve the differential equation $x^2y' + \frac{1}{y^2} = 0$, given the initial condition y(1)=2. (15 points)

$y^2 \cancel{\frac{x^2}{x^2}} y' = \frac{-1}{\cancel{y^2}} \frac{1}{x^2} \cdot \cancel{y^2}$

$\int y^2\,dy = \int -x^{-2}\,dx$

$\frac{1}{3}y^3 = \frac{1}{x} + C$

$\frac{1}{3}(2)^3 = \frac{1}{1} + C$

$\frac{8}{3} - 1 = C$

$C = 5/3$

$\left(\frac{1}{3}y^3 = \frac{1}{x} + \frac{5}{3}\right)3$

$y^3 = \frac{3}{x} + 5$

$\boxed{y = \sqrt[3]{\frac{3}{x} + 5}}$

6. Find the indicated partial derivatives for the function $f(x,y,z) = 2xy^2z^6 - e^{3-x}\ln(y-z)$. (10 points each)

a. $f_x = 2y^2z^6 - e^{3-x}(-1)\ln(y-z)$

$$\boxed{2y^2z^6 + e^{3-x}\ln(y-z)}$$

b. $f_y = 4xyz^6 - e^{3-x}\frac{1}{y-z}\cdot 1$

c. $f_z = 12xy^2z^5 - e^{3-z}\frac{1}{y-z}\cdot(-1)$

$$\boxed{12xy^2z^5 + e^{3-z}\frac{1}{y-z}}$$

d. $f_{xy} = 4yz^6 + e^{3-x}\frac{1}{y-z}\cdot 1$

$(y-z)^{-1}$

e. $f_{xyz} = 24yz^5 + e^{3-x}(y-z)^{-2}\cancel{(-1)(-1)}$

$$= \boxed{24yz^5 + \frac{e^{3-x}}{(y-z)^2}}$$

7. For the two demand equations $q_A = \frac{100}{p_A\sqrt{p_B}}$ and $q_B = 500/p_B\sqrt[3]{p_A}$, determine if the products are competitive, complementary or neither. (15 points)

$$q_A = \frac{100}{p_A} p_B^{-1/2} \qquad \frac{\partial q_A}{\partial p_B} = \frac{100}{p_A}\left(-\tfrac{1}{2}\right) p_B^{-3/2} < 0$$

$$q_B = \frac{500}{p_B} \cdot p_A^{-1/3} \qquad \frac{\partial q_B}{\partial p_A} = \frac{500}{p_B}\left(-\tfrac{1}{3}\right) p_A^{-4/3} < 0$$

$\boxed{\text{Complementary}}$

8. Find $\frac{\partial z}{\partial x}$ for $e^{yz} = -xyz$ implicitly. You must show the long way, but you may check your work by means of the short-cut formula. (15 points)

$$e^{yz} \cdot y z_x = -yz - xyz_x$$

$$e^{yz} y z_x + xyz_x = -yz$$

$$z_x\left(ye^{yz} + xy\right) = -yz$$

$$\boxed{\frac{\partial z}{\partial x} = z_x = \frac{-yz}{ye^{yz} + xy}}$$

check

$F(x, y, z) = e^{yz} + xyz$

$F_x = yz$

$F_z = ye^{yz} + xy$

$$\frac{-F_x}{F_z} = \frac{-yz}{ye^{yz} + xy} \checkmark$$

9. An investment of $3000 earns interest at an annual interest rate of 5% compounded continuously. After t years, its value S (in dollars) is given by $S = 3000e^{0.05t}$. Find the average value of a two-year investment. (10 points)

$$\int_0^2 3000e^{.05t}\,dt =$$

$$\frac{3000e^{.05t}}{.05}\Big|_0^2 = \frac{3000e^{.05(2)}}{.05} - \frac{3000e^{0}}{.05} =$$

$$\frac{3000}{.05}(e^{.1}-1) = 60{,}000(e^{.1}-1) \approx$$

$$\$6{,}310.26$$

(total)

$$\text{average} = \frac{1}{2}\int_0^2 3000e^{.05t}\,dt = \frac{6310.26}{2} =$$

$$\boxed{\$3{,}155.13}$$