Math B1131, Exam #2, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

Part 1. Integration by Tables.

1. Integrate using the attached table. Be sure to state the number of the equation used, and then use the formula to complete the problem. Indicate any substitutions needed to match the equation. (10 points each)

a. $\int \frac{5}{x^2(7-2x)}dx$ $x=u$ $du=dx$ $a=7,\ b=-2$ $5\int \frac{dx}{x^2(7-2x)}$

#6:

$$\int \frac{du}{u^2(a+bu)} = \frac{-1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$5\left[\frac{-1}{7x} - \frac{2}{49}\ln\left|\frac{7-2x}{x}\right|\right] + C$$

b. $\int \frac{x^2}{(4x^2-1)^{3/2}}dx$ $u=2x$ $du=2dx$ $\frac{1}{8}\int \frac{4x^2\cdot 2x\,dx}{(4x^2-1)^{3/2}}$

#33 $$\int \frac{u^2\,du}{(u^2\pm a^2)^{3/2}} = \frac{-u}{\sqrt{u^2\pm a^2}} + \ln\left|u+\sqrt{u^2\pm a^2}\right| + C$$

$$\frac{1}{8}\left[\frac{-2x}{\sqrt{4x^2-1}} + \ln\left|2x+\sqrt{4x^2-1}\right|\right] + C$$

c. $\int \frac{dx}{4-3e^{-2x}}$ $a=4,\ b=-3,\ c=-2$ $u=x$ $du=dx$

#45 $$\int \frac{du}{a+be^{cu}} = \frac{1}{ac}\left(cu - \ln|a+be^{cu}|\right) + C$$

$$= \left(\frac{-1}{8}\cdot\left(-2x - \ln|4-3e^{-2x}|\right)\right) + C$$

APPENDIX

B

Table of Selected Integrals

Rational Forms Containing $(a + bu)$

1. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

2. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C$

3. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C$

4. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C$

5. $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

6. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

7. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left(\ln|a+bu| + \frac{a}{a+bu}\right) + C$

8. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C$

9. $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

10. $\displaystyle\int \frac{du}{u^2(a+bu)^2} = -\frac{a+2bu}{a^2u(a+bu)} + \frac{2b}{a^3}\ln\left|\frac{a+bu}{u}\right| + C$

11. $\displaystyle\int \frac{du}{(a+bu)(c+ku)} = \frac{1}{bc-ak}\ln\left|\frac{a+bu}{c+ku}\right| + C$

12. $\displaystyle\int \frac{u\,du}{(a+bu)(c+ku)} = \frac{1}{bc-ak}\left[\frac{c}{k}\ln|c+ku| - \frac{a}{b}\ln|a+bu|\right] + C$

Forms Containing $\sqrt{a+bu}$

13. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C$

14. $\displaystyle\int u^2\sqrt{a+bu}\,du = \frac{2(8a^2-12abu+15b^2u^2)(a+bu)^{3/2}}{105b^3} + C$

15. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C$

16. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C$

17. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}} \right| + C, \quad a > 0$

18. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$

Forms Containing $\sqrt{a^2 - u^2}$

19. $\displaystyle\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

20. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$

21. $\displaystyle\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2 u} + C$

22. $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u} = \sqrt{a^2-u^2} - a \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C, \quad a > 0$

Forms Containing $\sqrt{u^2 \pm a^2}$

23. $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2 \ln \left| u + \sqrt{u^2 \pm a^2} \right| \right) + C$

24. $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - \frac{a^4}{8} \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

25. $\displaystyle\int \frac{\sqrt{u^2+a^2}\,du}{u} = \sqrt{u^2+a^2} - a \ln \left| \frac{a+\sqrt{u^2+a^2}}{u} \right| + C$

26. $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}\,du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

27. $\displaystyle\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

28. $\displaystyle\int \frac{du}{u\sqrt{u^2+a^2}} = \frac{1}{a} \ln \left| \frac{\sqrt{u^2+a^2}-a}{u} \right| + C$

29. $\displaystyle\int \frac{u^2\,du}{\sqrt{u^2 \pm a^2}} = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2 \ln \left| u + \sqrt{u^2 \pm a^2} \right| \right) + C$

30. $\displaystyle\int \frac{du}{u^2\sqrt{u^2 \pm a^2}} = -\frac{\pm\sqrt{u^2 \pm a^2}}{a^2 u} + C$

31. $\displaystyle\int (u^2 \pm a^2)^{3/2} du = \frac{u}{8}(2u^2 \pm 5a^2)\sqrt{u^2 \pm a^2} + \frac{3a^4}{8} \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

32. $\displaystyle\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$

33. $\displaystyle\int \frac{u^2\,du}{(u^2 \pm a^2)^{3/2}} = \frac{-u}{\sqrt{u^2 \pm a^2}} + \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

Rational Forms Containing $a^2 - u^2$ and $u^2 - a^2$

34. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$

35. $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

Exponential and Logarithmic Forms

36. $\int e^u du = e^u + C$

37. $\int a^u du = \dfrac{a^u}{\ln a} + C, \quad a > 0, a \neq 1$

38. $\int ue^{au} du = \dfrac{e^{au}}{a^2}(au - 1) + C$

39. $\int u^n e^{au} du = \dfrac{u^n e^{au}}{a} - \dfrac{n}{a}\int u^{n-1} e^{au}\, du$

40. $\int \dfrac{e^{au} du}{u^n} = -\dfrac{e^{au}}{(n-1)u^{n-1}} + \dfrac{a}{n-1}\int \dfrac{e^{au} du}{u^{n-1}}, \quad n \neq 1$

41. $\int \ln u\, du = u \ln u - u + C$

42. $\int u^n \ln u\, du = \dfrac{u^{n+1} \ln u}{n+1} - \dfrac{u^{n+1}}{(n+1)^2} + C, \quad n \neq -1$

43. $\int u^n \ln^m u\, du = \dfrac{u^{n+1}}{n+1} \ln^m u - \dfrac{m}{n+1}\int u^n \ln^{m-1} u\, du, \quad m, n \neq -1$

44. $\int \dfrac{du}{u \ln u} = \ln \left| \ln u \right| + C$

45. $\int \dfrac{du}{a + be^{cu}} = \dfrac{1}{ac}\left(cu - \ln \left| a + be^{cu} \right|\right) + C$

Miscellaneous Forms

46. $\int \sqrt{\dfrac{a+u}{b+u}} du = \sqrt{(a+u)(b+u)} + (a-b) \ln \left(\sqrt{a+u} + \sqrt{b+u}\right) + C$

47. $\int \dfrac{du}{\sqrt{(a+u)(b+u)}} = \ln \left| \dfrac{a+b}{2} + u + \sqrt{(a+u)(b+u)} \right| + C$

48. $\int \sqrt{a + bu + cu^2} du = \dfrac{2cu + b}{4c}\sqrt{a + bu + cu^2}$

$$- \frac{b^2 - 4ac}{8c^{3/2}} \ln \left| 2cu + b + 2\sqrt{c}\sqrt{a + bu + cu^2} \right| + C, \quad c > 0$$