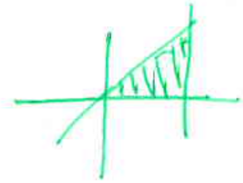
Math 2153, Quiz #9, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the volume of the solid bounded by the graphs of the equations. Sketch the region in the plane that the integral is defined over.
   a. The region between $z = x + y$ and the x-y plane, over the region $y = x, x = 5$ and in the first quadrant.

$$\int_0^5 \int_0^x x+y \, dy\, dx$$

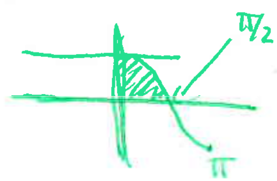


$$\int_0^5 \tfrac{1}{2}y^2 + xy \Big|_0^x = \int_0^5 \tfrac{1}{2}x^2 + x^2 \, dx =$$

$$\int_0^5 \tfrac{3}{2}x^2 \, dx = \tfrac{1}{2}x^3 \Big|_0^5 = \boxed{\tfrac{125}{2}}$$

   b. $\int_0^1 \int_0^{arccosy} \sin(x)\sqrt{1 + sin^2(x)}dxdy$ [Hint: You might need to switch the order of integration.]

$x = \arccos y \Rightarrow y = \cos x$ no more than $[0, \pi]$



$$\int_0^{\pi/2} \int_0^{\cos x} \sin x \sqrt{1+\sin^2 x} \, dy\,dx = \int_0^{\pi/2} y \sin x \sqrt{1+\sin^2 x} \Big|_0^{\cos x} dx$$

$$\int_0^{\pi/2} \cos x \sin x \sqrt{1+\sin^2 x} \, dx \rightarrow \int \tfrac{1}{2} u^{1/2} du = \tfrac{1}{\cancel{2}} \cdot \tfrac{\cancel{2}}{3} u^{3/2} \rightarrow$$

$u = \sin^2 x + 1$

$\tfrac{1}{2} du = \cancel{2} \sin x \cos x \, dx$

$$\tfrac{1}{3}(1+\sin^2 x)^{3/2} \Big|_0^{\pi/2} = \boxed{\tfrac{1}{3}\left[2^{3/2} - 1\right]}$$

$$\frac{2\sqrt{2}-3}{3}$$

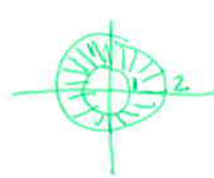
   c. The region under $z = \ln(x^2 + y^2)$ and z=0, over the region bounded by $x^2 + y^2 \geq 1$ and $x^2 + y^2 \leq 4$.

$z = \ln r^2 = 2\ln r$

$$\int_0^{2\pi} \int_1^2 r \cdot 2\ln r \, dr\, d\theta$$

$u = \ln r \quad dv = 2r$

$u = \tfrac{1}{r} dr \quad v = r^2$



$$\int_0^{2\pi} \left[r^2 \ln r - \int_1^2 \frac{r^2}{r} dr \right] d\theta = \int_0^{2\pi} r^2 \ln r - \tfrac{1}{2} r^2 \Big|_1^2 d\theta =$$

$$\frac{r^2}{r} \Downarrow r$$

$$\int_0^{2\pi} \cancel{2}4\ln 2 - 2 + \tfrac{1}{2} \, d\theta = \boxed{2\pi\left[4\ln 2 - \tfrac{3}{2}\right]}$$