Math 2153, Quiz #8, Fall 2012 Name KEY

Instructions: Show all work. Use exact values unless specifically asked to round.

1. Use Lagrange Multipliers to find any extrema of the function $f(x, y) = x^2 + 3xy + y^2$ subject to the constraint $x^2 + y^2 \leq 1$.

$F(x,y,\lambda) = x^2 + 3xy + y^2 - \lambda(x^2 + y^2 - 1)$

$F_x = 2x + 3y - 2\lambda x = 0$ $\quad \frac{2x+3y}{x} = 2\lambda$

$F_y = 3x + 2y - 2\lambda y = 0$ $\quad \frac{3x+2y}{y} = 2\lambda$

$\Rightarrow \frac{2x+3y}{x} = \frac{3x+2y}{y}$

$\cancel{2xy} + 3y^2 = 3x^2 + \cancel{2xy}$

$3y^2 = 3x^2$

$y^2 = x^2$

$x = y$

or $x = -y$

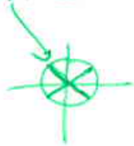
$x^2 + y^2 = 1$

$x^2 + x^2 = 1$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

extrema $x = \pm\frac{1}{\sqrt{2}}$ $\quad y = \pm\frac{1}{\sqrt{2}}$

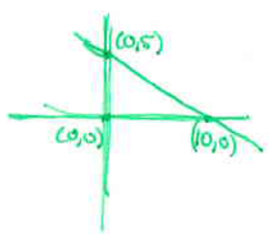


$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

these are all on boundary $x^2 + y^2 = 1$

since condition is $x^2 + y^2 \leq 1$ another extrema that satisfies $y^2 = x^2$ is also $(0,0)$ and any point on lines $y = x$ & $y = -x$

2. Find the absolute extrema of the function $f(x, y) = x^2 + y^2$ on the region $R: \left\{(x, y) | x \geq 0, y \geq 0, y \leq -\frac{1}{2}x + 5\right\}$. Sketch the region.



① $f_x = 2x = 0$ $\quad x = 0$ $\quad (0,0)$

$f_y = 2y = 0$ $\quad y = 0$

③ corners

$(0,0)$, $(0,5)$, $(10,0)$

used.

② boundaries

a) $x = 0$

$f(0,y) = y^2$

$f'(0,y) = 2y = y = 0$ $\quad (0,0)$ used

b) $y = 0$

$f(x,0) = x^2$

$f'(x,0) = 2x = 0$

$x = 0$

$(0,0)$ used

c) $f(x, -\frac{1}{2}x + 5) = x^2 + (-\frac{1}{2}x + 5)^2$

$f'(x, -\frac{1}{2}x + 5) = 2x + (-\frac{1}{2}x + 5)(-\frac{1}{2})\cancel{2}$

$2x + \frac{1}{2}x - 5 = 0$

$\frac{2}{5} \cdot \frac{5}{2}x = 5 \cdot \frac{2}{5}$

$x = 2$

$y = -\frac{1}{2}(2) + 5$

$y = -1 + 5 = 4$

$(2,4)$

Pts.

$(0,0)$

$(0,5)$

$(10,0)$

$(2,4)$

$f(0,0) = 0$

$f(0,5) = 25$

$f(10,0) = 100$

$f(2,4) = 4 + 16 = 20$

$f(0,0)$ MIN

$f(10,0) =$ MAX