Math 2153, Quiz #6, Fall 2012   Name KEY

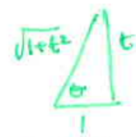
Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find $\frac{dw}{dt}$ for $w = x\sec(y) + xy\ln(y), x = 2\sin(t)\, e^t, y = \arctan(t) - t$ by the chain rule. Be sure to simplify any expressions like $\sec(\arctan(t))$ or the like.

$$\frac{\partial w}{\partial x} = \sec y + y\ln y = \sec(\overset{\theta}{\arctan t} - t) = \frac{1}{\cos\theta\cos t + \sin\theta\sin t} + (\arctan t - t)\cdot\ln(\arctan t - t)$$

$$\frac{\partial w}{\partial y} = x\sec y\tan y + x\ln y + xy\cdot\frac{1}{y} = 2\sin(t)e^t \frac{1}{\cos\theta\cos t + \sin\theta\sin t}\cdot\frac{\tan\theta - \tan t}{1+\tan\theta\tan t} + 2\sin(t)e^t\ln(\arctan t - t) + 2e^t\sin t$$

$\tan\theta = t$



$\cos\theta = \frac{1}{\sqrt{1+t^2}}$

$\sin\theta = \frac{t}{\sqrt{1+t^2}}$

$$\frac{dx}{dt} = 2e^t\sin t + 2\cos(t)(e^t)$$

$$\frac{dy}{dt} = \frac{1}{1+t^2} - 1$$

$$\frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} = \left[\frac{1}{\frac{1}{\sqrt{1+t^2}}\cos t + \frac{t}{\sqrt{1+t^2}}\sin t} + (\arctan t - t)\ln(\arctan t - t)\right](2e^t)(\sin t + \cos t)$$
$$+ \left[\frac{2e^t\sin t\,(t - \tan t)}{\left(\frac{1}{\sqrt{1+t^2}}\cos t + \frac{t}{\sqrt{1+t^2}}\sin t\right)(1 + t\tan t)} + 2e^t\sin t\ln(\arctan t - t) + 2e^t\sin t\right]\left(\frac{1}{1+t^2} - 1\right)$$

2. Find $\frac{\partial w}{\partial y}$ for $xyz + xzw - yzw + w^2 = 5$ by any means.

$F(x,y,z,w) = xyz + xzw - yzw + w^2 - 5$

$$\frac{\partial w}{\partial y} = -\frac{F_y}{F_w} = -\frac{xz + 0 - zw}{0 + xz - yz + 2w} = \boxed{\frac{zw - xz}{xz - yz + 2w}}$$

3. Find the value of the directional derivative for the function $f(x,y,z) = xyz$, at the point P(2,3,5) in the direction $\vec{v} = <2,1,2>$.

$$\nabla f = \langle yz, xz, xy\rangle = \langle 15, 10, 6\rangle$$

$$\|\vec{v}\| = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$\hat{u} = \langle 2/3, 1/3, 2/3\rangle$$

$$\nabla f\cdot\hat{u} = \langle 15, 10, 6\rangle\cdot\langle 2/3, 1/3, 2/3\rangle = 10 + 10/3 + 4 = \frac{30+10+12}{3} = \boxed{\frac{52}{3}}$$