Math 2153, Quiz #4, Fall 2012 Name KEY

Instructions: Show all work. Give exact values unless specifically asked to round.

1. Find the curvature of the vector-valued function $\vec{r}(t) = 4\sin(t)\,\hat{\imath} - 4\cos(t)\,\hat{\jmath} + 2\sin(t)\hat{k}$. Find the value(s) of t, and the corresponding point(s) in (x,y,z) coordinates where the curvature is a maximum. (You may find the maximum point(s) numerically.)

$r'(t) = 4\cos t\,\hat{\imath} + 4\sin t\,\hat{\jmath} + 2\cos t\,\hat{k}$

$r''(t) = -4\sin t\,\hat{\imath} + 4\cos t\,\hat{\jmath} - 2\sin t\,\hat{k}$

$$r' \times r'' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4\cos t & 4\sin t & 2\cos t \\ -4\sin t & 4\cos t & -2\sin t \end{vmatrix} =$$

$$(-8\sin^2 t - 8\cos^2 t)\hat{\imath} - (-8\sin t\cos t + 8\sin t\cos t)\hat{\jmath} + (16\cos^2 t + 16\sin^2 t)\hat{k}$$

$$-8\,\hat{\imath} \qquad 0\,\hat{\jmath} \qquad + 16\,\hat{k}$$

$\|r' \times r''\| = \sqrt{64 + 256} = \sqrt{320} = 8\sqrt{5}$

$\|r'\| = \sqrt{\cancel{16}\cos^2 t + 16\sin^2 t + 4\cos^2 t} = \sqrt{16 + 4\cos^2 t} = 2\sqrt{4 + \cos^2 t}$

$K = \dfrac{8\sqrt{5}}{(2\sqrt{4+\cos^2 t})^3} = \dfrac{\sqrt{5}}{(\sqrt{4+\cos^2 t})^3}$ Max at $t = \pi/2$ (& odd multiple of $\pi/2$)

in space $(4, 0, 2)$