Math 2153, Quiz #3, Fall 2012 Name KEY

Instructions: Show all work. Give exact values unless specifically asked to round.

1. Find the unit tangent vector, and the unit normal vector to the vector-valued function $\vec{r}(t) = 4t\hat{\imath} + \cos(5t)\hat{\jmath} - \sin(5t)\hat{k}$.

$r'(t) = 4\hat{\imath} + -5\sin 5t\,\hat{\jmath} - 5\cos 5t\,\hat{k}$

$\|r'(t)\| = \sqrt{16 + 25\sin^2 5t + 25\cos^2 5t} = \sqrt{16+25} = \sqrt{41}$

$\vec{T}(t) = \frac{1}{\sqrt{41}}\left[4\hat{\imath} - 5\sin 5t\,\hat{\jmath} - 5\cos 5t\,\hat{k}\right]$

$\vec{N}(t) = \frac{T'(t)}{\|T'(t)\|}$. $T'(t) = \frac{1}{\sqrt{41}}\left[0\hat{\imath} - 25\cos 5t\,\hat{\jmath} + 25\sin 5t\,\hat{k}\right]$

$\|T'(t)\| = \frac{1}{\sqrt{41}}\sqrt{625\cos^2 \cancel{5t} + 625\sin^2 t} = \frac{1}{\sqrt{41}}\sqrt{625} = \frac{25}{\sqrt{41}}$

$\vec{N}(t) = -\cos 5t\,\hat{\jmath} + \sin 5t\,\hat{k}$

2. Find the arc length of $\vec{r}(t) = 4\sin(t)\,\hat{\imath} - 4\cos(t)\,\hat{\jmath} + 2\sin(t)\hat{k}$. Set up the integral and then complete it numerically in your calculator. Round to 4 decimal places.

pick an interval since I didn't specify I'll use $[0, 2\pi]$ for numerical answer here.

$r'(t) = 4\cos t\,\hat{\imath} + 4\sin t\,\hat{\jmath} + 2\cos t\,\hat{k}$

$\|r'(t)\| = \sqrt{16\cos^2 t + 16\sin^2 t + 4\cos^2 t} =$

$\sqrt{16 + 4\cos^2 t} = 2\sqrt{4 + \cos^2 t}$

$\int_0^{2\pi} 2\sqrt{4+\cos^2 t}\, dt \approx 26.636\ldots$