Math 2153, Quiz #16, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Evaluate the surface integral $\int_S \int (x - 2y + z)dS$ on the surface $S: z = 15 - 2x + 3y, 0 \le x \le 2, 0 \le y \le 4$.

$dS = \sqrt{1+4+9}\, dA = \sqrt{14}\, dA$

$\int_0^2 \int_0^4 (x-2y+15-2x+3y)\sqrt{14}\, dy\, dx$

$\int_0^2 \int_0^4 15-x+y)\, dy\, dx = \int_0^2 15y - xy + \frac{1}{2}y^2 \Big|_0^4 dx =$

$\int_0^2 60-4x+8\, dx = \int_0^2 68-4x\, dx = 68x-2x^2\Big|_0^2 =$

$136-8 = \boxed{128}$

2. Use the Divergence Theorem to evaluate $\int_S \int \vec{F}\cdot\vec{N} dS$ to find the outward flux of $\vec{F}(x,y,z) = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ through the surface of the solid bounded by the graphs of $S: x = 0, x = a, y = 0, y = a, z = 0, z = a$.

$\operatorname{div} F = 2x+2y+2z$

$\int_0^a \int_0^a \int_0^a 2x+2y+2z\, dz\, dy\, dx =$

$\int_0^a \int_0^a 2xz+2yz+z^2\Big|_0^a dy\, dx = \int_0^a \int_0^a 2xa+2ya+a^2\, dy\, dx$

$= \int_a^a 2axy+y^2a+a^2y\Big|_0^a dx = \int_0^a 2a^2x+a^3+a^3\, dx = \int_0^a 2a^2x+2a^3 dx$

$= a^2x^2+2a^3x\Big|_0^a = a^4+2a^4 = \boxed{3a^4}$