Math 2153, Quiz #13, Fall 2012 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round.

1. Determine if the vector field $\vec{F}(x,y) = \frac{x\vec{\imath}+y\vec{\jmath}}{x^2+y^2}$ is conservative. If it is, find the potential function.

$$\frac{\partial M}{\partial y} = x(x^2+y^2)^{-1} = x(-1)(x^2+y^2)^{-2}(2y) = \frac{-2xy}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = y(x^2+y^2)^{-1} = y(-1)(x^2+y^2)^{-2}(2x) = \frac{-2xy}{(x^2+y^2)^2}$$

✓ is conservative

$$\int \frac{x}{x^2+y^2}\,dx = \frac{1}{2}\ln|x^2+y^2| + C(y)$$

$$\int \frac{y}{x^2+y^2}\,dy = \frac{1}{2}\ln|x^2+y^2| + C(x)$$

$$\boxed{f(x,y) = \frac{1}{2}\ln|x^2+y^2| + K}$$

2. Find the curl of the vector field $\vec{F}(x,y,z) = ye^z\vec{\imath} + xe^z\vec{\jmath} + xye^z\vec{k}$.

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ ye^z & xe^z & xye^z \end{vmatrix} = (xe^z - xe^z)\hat{\imath} - (ye^z - ye^z)\hat{\jmath} + (e^z - e^z)\hat{k} = \boxed{\vec{0}}$$

3. Find the divergence of $\vec{F}(x,y,z) = x^2y\vec{\imath} - 2xz\vec{\jmath} + yz\vec{k}$.

$$\vec{\nabla}\cdot\vec{F} = 2xy - 0 + y = \boxed{2xy + y}$$