Math 2153, Quiz #11, Fall 2012 Name KEY

Instructions: Show all work. Give exact answers unless otherwise told to round.

1. Set up a triple integral to find the volume of the solid bounded by $z = x, z = 0, x = 4 - y^2$. Find the volume of the solid.

$$\int_{-2}^{2}\int_{0}^{4-y^2}\int_{0}^{x} 1\, dz\, dx\, dy$$

$$z\Big|_0^x = x$$

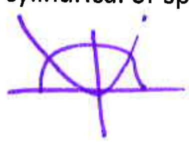
$$\int_{-2}^{2}\int_{0}^{4-y^2} x\, dx\, dy = \int_{-2}^{2} \frac{1}{2}x^2\Big|_0^{4-y^2} dy = \int_{-2}^{2}\frac{1}{2}(4-y^2)^2\, dy$$

$$\frac{1}{2}\int_{-2}^{2} 16 - 8y^2 + y^4\, dy = \frac{1}{2}\left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right|_{-2}^{2} = \frac{1}{2}\left[32 - \frac{64}{3} + \frac{32}{5}\right]\times 2$$

$$= \boxed{256/15}$$

2. Set up a triple integral to find the volume of the solid that is bounded by the graphs of $x^2 + y^2 + z^2 = 80, z = \frac{1}{2}(x^2 + y^2)$. (The interior common to both graphs.) Find the volume of the solid. [Hint: it may help to switch to cylindrical or spherical.]

$z = \sqrt{80 - r^2}$ $z = \frac{1}{2}r^2$



$$\frac{1}{4}r^4 = 80 - r^2$$

$$\frac{1}{4}r^4 + r^2 - 80 = 0$$

$$r^4 + 4r^2 - 320 = 0$$

$$(r^2 + 20)(r - 16) = 0$$

$$r^2 = \cancel{-20}, 16$$

$$r = 4$$

$$\int_0^{2\pi}\int_0^{4}\int_{\frac{1}{2}r^2}^{\sqrt{80-r^2}} r\, dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{4} z\, r\Big|_{\frac{1}{2}r^2}^{\sqrt{80-r^2}} dr\, d\theta =$$

$$\int_0^{2\pi}\int_0^{4} r\sqrt{80-r^2} - \frac{1}{2}r^3\, dr\, d\theta = \int_0^{2\pi} -\frac{1}{2}\cdot\frac{2}{3}(80-r^2)^{3/2} - \frac{1}{8}r^4\Big|_0^4 =$$

$u = 80 - r^2$

$-\frac{1}{2}du = r\,dr$

$$\int_0^{2\pi} -\frac{1}{3}(512) + \frac{1}{3}\sqrt{80}^3 - 32\, d\theta = \boxed{\left(\frac{-608}{3} + \frac{1}{3}\sqrt{80}^3\right)2\pi}$$

$$\approx 225.23$$