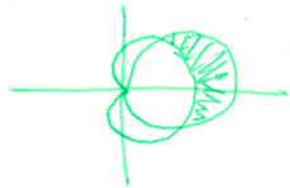
Math 2153, Quiz #10, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the center of mass of the planar region inside the graph of $r = 6\cos(\theta)$ and outside the graph of $r = 2 + 2\cos(\theta)$. The region's density is given by $\rho(r,\theta) = r\cos(\theta)$. [Your final answer will be in rectangular coordinates.]

integrate for r & then you can do numerically in θ, to 4 decimal places



$$6\cos\theta = 2 + 2\cos\theta$$
$$\frac{4\cos\theta}{2} = \frac{2}{2}$$
$$\cos\theta = 1/2$$
$$\theta = \pi/3,\ -\pi/3$$

$$M = \int_{-\pi/3}^{\pi/3}\int_{2+2\cos\theta}^{6\cos\theta} \underbrace{\overset{\rho}{r\cos\theta}\cdot r}_{r^2}\,\overset{dA}{dr\,d\theta}$$

$$\int_{-\pi/3}^{\pi/3} \frac{1}{3}r^3\Big|_{2+2\cos\theta}^{6\cos\theta}\cos\theta\,d\theta = \int_{-\pi/3}^{\pi/3}\frac{1}{3}\left[(6\cos\theta)^3 - (2+2\cos\theta)^3\right]\cos\theta\,d\theta = 53.8709$$

$$M_x = \int_{-\pi/3}^{\pi/3}\int_{2+2\cos\theta}^{6\cos\theta} \overbrace{\underset{x}{r\cos\theta}\cdot\underset{\rho}{r\cos\theta}\;r}^{r^3}\,dr\,d\theta = \int_{-\pi/3}^{\pi/3}\frac{1}{4}r^4\Big|_{2+2\cos\theta}^{6\cos\theta}\cos^2\theta\,d\theta =$$

$$\int_{-\pi/3}^{\pi/3}\frac{1}{4}\left[(6\cos\theta)^4 - (2+2\cos\theta)^4\right]\cos^2\theta\,d\theta = 241.7402$$

$$M_y = \int_{-\pi/3}^{\pi/3}\int_{2+2\cos\theta}^{6\cos\theta} \overbrace{\underset{y}{r\sin\theta}\;\underset{\rho}{r\cos\theta}\;r}^{r^3}\,dr\,d\theta = \int_{-\pi/3}^{\pi/3}\frac{1}{4}r^4\Big|_{2+2\cos\theta}^{6\cos\theta}\sin\theta\cos\theta\,d\theta$$

$$\int_{-\pi/3}^{\pi/3}\frac{1}{4}\left[(6\cos\theta)^4 - (2+2\cos\theta)^4\right]\sin\theta\cos\theta\,d\theta = 0$$

$$\frac{M_x}{M} = \bar{y} = \frac{241.7402}{53.8709} = 4.4874$$

$$\boxed{(\bar{x},\bar{y}) = (0,\ 4.4874)}$$

$$\frac{M_y}{M} = \bar{x} = \frac{0}{53.8709} = 0$$