Math 2153 Homework #2 Key

1) $3x + 2y - z = 7 \quad \times 2$ $\qquad$ $6x + \cancel{4y} - \cancel{2z} = 14$

$x - 4y + 2z = 0$ $\qquad\qquad\qquad$ $x - \cancel{4y} + \cancel{2z} = 0$

$$7x = 14$$

$$x = 2$$

$$\left.\begin{array}{ll} 6 + 2y - z = 7 \Rightarrow & 2y - z = 1 \\ 2 - 4y + 2z = 0 & -4y + 2z = -2 \end{array}\right\}$$ multiples of each other, thus

$$z = 2y - 1$$

$$\left.\begin{array}{l} x = 2 \\ y = t \\ z = 2t - 1 \end{array}\right\}$$ parametric equation

2) a) $Q(2,8,4)$ $\quad 2x + y + z = 5$ $\quad P(0,0,5)$ in plane

$\vec{PQ} = \langle 2, 8, -1 \rangle$ $\quad \vec{n} = \langle 2, 1, 1 \rangle$ $\quad \|\vec{n}\| = \sqrt{4+1+1} = \sqrt{6}$

distance: $\dfrac{\langle 2, 8, -1 \rangle \cdot \langle 2, 1, 1 \rangle}{\sqrt{6}} = \dfrac{4 + 8 - 1}{\sqrt{6}} = \dfrac{11}{\sqrt{6}}$

b) $Q(-2,1,3)$ $\quad x = 1 - t,\ y = 2 + t,\ z = -2t$ $\quad P(1,2,0)$ in plane

$\vec{PQ} : \langle -3, -1, 3 \rangle$ $\quad \vec{p} \langle -1, 1, -2 \rangle$ $\quad \|\vec{p}\| = \sqrt{1+1+4} = \sqrt{6}$

distance $= \dfrac{\| \langle -3, -1, 3 \rangle \times \langle -1, 1, -2 \rangle \|}{\sqrt{6}} = \dfrac{7\sqrt{2}}{\sqrt{6}} = \dfrac{7}{\sqrt{3}}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -1 & 3 \\ -1 & 1 & -2 \end{vmatrix} = (2-3)\hat{i} - (6+3)\hat{j} + (-3-1)\hat{k} = \langle -1, -9, -4 \rangle$$

$$\| \langle -1, -9, -4 \rangle \| = \sqrt{1 + 81 + 16} = \sqrt{98} = 7\sqrt{2}$$

3) a) $x^2 + y^2 + z^2 = 10$

Cylindrical $r^2 + z = 10$

Spherical $\rho = \sqrt{10}$

3 b) $y = x^2$

Cylindrical $\frac{r\sin\theta}{r} = \frac{r^2\cos^2\theta}{r} \Rightarrow \frac{\sin\theta}{\cos^2\theta} = r\frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$

$r = \sec\theta\tan\theta$

Spherical $\cancel{\rho}\sin\theta\,\cancel{\sin\varphi} = \cancel{\rho^2}\,\rho\,\sin^{\cancel{2}}\varphi\cos^2\theta$

$$\frac{\sin\theta}{\sin\varphi\cos^2\theta} = \rho\frac{\sin\varphi\cos^2\theta}{\sin\varphi\cos^2\theta}$$

$\rho = \csc\varphi\sec\theta\tan\theta$

c) $x^2 + y^2 = 9$

Cylindrical $r = 3$

Spherical $\rho^2\sin^2\varphi = 9 \Rightarrow \rho = 3\csc\varphi$

d) $x = 4$

Cylindrical $r\cos\theta = 4 \Rightarrow r = 4\sec\theta$

Spherical $\rho\sin\varphi\cos\theta = 4 \Rightarrow \rho = 4\csc\varphi\sec\theta$

e) $x^2 + y^2 - 3z^2 = 0$

Cylindrical

$r^2 - 3z^2 = 0 \Rightarrow 3z^2 = r^2 \Rightarrow z^2 = \frac{1}{3}r^2 \Rightarrow z = \frac{\pm 1}{\sqrt{3}}r$

Spherical

$\cancel{\rho^2}\sin^2\varphi - 3\cancel{\rho^2}\cos^2\varphi = 0 \Rightarrow \sin^2\varphi = 3\cos^2\varphi \Rightarrow 3 = \tan^2\varphi$

$\Rightarrow \sqrt{3} = \tan\varphi$

4) a) $r = 2 \Rightarrow r^2 = 4 \Rightarrow x^2 + y^2 = 4$

b) $r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2 + y^2 = 2y \Rightarrow x^2 + y^2 - 2y + 1 = 0 + 1$

$x^2 + (y-1)^2 = 1$

4c) $z = r^2 \cos^2\theta \Rightarrow z = x^2$

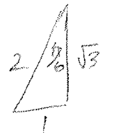
d) $r = \frac{1}{2}z \Rightarrow z = 2r \Rightarrow z^2 = 4r^2 \Rightarrow 4x^2 + 4y^2 - z^2 = 0$

e) $\rho = 2\sec\varphi \Rightarrow \rho\cos\varphi = 2 \Rightarrow z = 2$

f) $\rho = 4\csc\varphi\sec\theta \Rightarrow \rho\sin\varphi\cos\theta = 4 \Rightarrow x = 4$

g) $\rho = 2 \Rightarrow \rho^2 = 4 \Rightarrow x^2 + y^2 + z^2 = 4$

h) $\varphi = \frac{\pi}{6}$

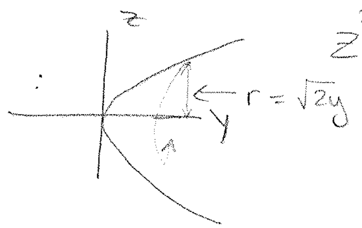


$\tan\varphi = \frac{1}{\sqrt{3}} \Rightarrow \sin\varphi = \frac{1}{\sqrt{3}}\cos\varphi \Rightarrow \sin^2\varphi = \frac{1}{3}\cos^2\varphi$

$\Rightarrow \rho^2\sin^2\varphi = \frac{1}{3}\rho^2\cos^2\varphi \Rightarrow x^2 + y^2 = \frac{1}{3}z^2$

$\Rightarrow 3x^2 + 3y^2 - z^2 = 0$

5)



$z^2 = 2y \qquad z = \sqrt{2y} = [r(y)]$

revolved around y-axis:

$x^2 + z^2 = [r(y)]^2$

$x^2 + z^2 = (\sqrt{2y})^2$

$x^2 + z^2 = 2y$

6) each "hour" represents $\frac{360}{24} = 15°$

$14^h 36^m 36^s = 14.66$ hours $= 219.9°$

Converting to radians: $219.9 * \frac{\pi}{180} = \frac{733}{600}\pi \approx 3.837979$ radians

$-60°50'14''$ is south from the equator $\approx 60.837\overline{2}°$

$90 + 60.837\overline{2} \approx 150.837\overline{2}°$ from north polar axis (i.e. +z axis)

$150.837\overline{2} \cdot \frac{\pi}{180} \approx .837984\ldots\pi \approx 2.6326$ radians

in spherical coordinates then $(4.366, 2.6326, 3.8380)$

(ρ, φ, θ)

w/ ρ in light years and angles in radians

6) continued

in rectangular coordinates

$X = \rho\cos\theta\sin\varphi = 4.366\cos(3.8380)\sin(2.6326) = -1.6321$

$Y = \rho\sin\theta\sin\varphi = 4.366\sin(3.8380)\sin(2.6326) = -1.3647$

$z = \rho\cos\varphi = 4.366\cos(2.6326) = -3.8125$

$(x, y, z) = (-1.6321, -1.3647, -3.8125)$

w/ x, y, z in light years

7) a) $\vec{r}(t) = \sqrt{4-t^2}\,\hat{i} + t^2\hat{j} - 6t\,\hat{k}$

$[-2,2] \cap (-\infty,\infty) \cap (-\infty,\infty) \Rightarrow D: [-2,2]$

$\|\vec{r}(t)\| = \sqrt{(\sqrt{4-t^2})^2 + (t^2)^2 + (-6t)^2} = \sqrt{4-t^2+t^4+36t^2} = \sqrt{t^4+35t^2+4}$

b) $\vec{r}(t) = (\ln t - 1)\hat{i} + t\hat{j}$

$(0,\infty) \cap (-\infty,\infty) \Rightarrow D: (0,\infty)$

$\|\vec{r}(t)\| = \sqrt{(\ln t-1)^2 + t^2} =$

c) $\vec{r}(t) = 3\cos t\,\hat{i} + 2\sin t\,\hat{j} + t^2$

$(-\infty,\infty) \cap (-\infty,\infty) \cap (-\infty,\infty) \Rightarrow D: (-\infty,\infty)$

$\|\vec{r}(t)\| = \sqrt{9\cos^2 t + 4\sin^2 t + t^4} = \sqrt{4\cos^2 t + 5\cos^2 t + 4\sin^2 t + t^4} =$

$\sqrt{4 + 5\cos^2 t + t^4}$

d) $\vec{r}(t) = \sqrt[3]{t}\,\hat{i} + \frac{1}{t+1}\hat{j} + (t+2)\hat{k}$

$(-\infty,\infty) \cap [(-\infty,-1)\cup(-1,\infty)] \cap (-\infty,\infty) \Rightarrow D: (-\infty,-1)\cup(-1,\infty)$

$\|\vec{r}(t)\| = \sqrt{t^{2/3} + \frac{1}{(t+1)^2} + (t+2)^2} = \dfrac{\sqrt{t^{2/3}(t+1)^2 + 1 + (t^2+3t+2)^2}}{t+1}$

7 e) $\vec{r}(t) = (1-t)\hat{\imath} + \sqrt{t}\,\hat{k}$

$(-\infty,\infty) \cap [0,\infty) \Rightarrow D: [0,\infty)$

$\|\vec{r}(t)\| = \sqrt{(1-t)^2 + (\sqrt{t})^2} = \sqrt{1-2t+t^2+t} = \sqrt{t^2-t+1}$

8) a) $y = 4-x$ i) $x=t$ $y=4-t$ ii) $x=e^t$ $y=4-e^t$

$\vec{r}_1(t) = t\hat{\imath} + (4-t)\hat{\jmath}$

$\vec{r}_2(t) = e^t\hat{\imath} + (4-e^t)\hat{\jmath}$

choose x to be some function of t, solve for y

b) $x^2+y^2=25$ i) $x = r\cos\theta$, $y = r\sin\theta$

$= 5\cos t$ $= 5\sin t$

$\vec{r}_1(t) = 5\cos t\,\hat{\imath} + 5\sin t\,\hat{\jmath}$ (standard clockwise orientation)

$\vec{r}_2(t) = 5\sin t\,\hat{\imath} + 5\cos t\,\hat{\jmath}$ (switching changes to clockwise orientation)

c) $y = 4-x^2$ i) $x=t$ ii) $x=3t$ $y=4-9t^2$

$\vec{r}_1(t) = t\hat{\imath} + (4-t^2)\hat{\jmath}$

$\vec{r}_2(t) = 3t\hat{\imath} + (4-9t^2)\hat{\jmath}$

9) a) $z = x^2+y^2$, $x+y=0$, $x=t$ $\vec{r}(t) = t\hat{\imath} - t\hat{\jmath} + 2t^2\hat{k}$

$t+y=0 \Rightarrow y=-t$

$z = t^2+(-t)^2 \Rightarrow z = 2t^2$

b) $x^2+y^2+z^2=4$, $x+z=2$; $x=1+\sin t$ $1+\sin t + z = 2 \Rightarrow z = 1-\sin t$

$(1+\sin t)^2+y^2+(1-\sin t)^2=4 \Rightarrow 1+\cancel{2\sin t}+\sin^2 t + y^2 + 1 - \cancel{2\sin t} + \sin^2 t = 4$

$y^2 = 2-2\sin^2 t = \sqrt{2}\cos t$ $\vec{r}(t) = (1+\sin t)\hat{\imath} + \sqrt{2}\cos t\,\hat{\jmath} + (1-\sin t)\hat{k}$

c) $4x^2+4y^2+z^2=16$, $x=z^2$, $z=t$ $x=t^2 \Rightarrow 4t^4+4y^2+t^2=16$

$4y^2 = 16-4t^4-t^2 \Rightarrow y = \pm\frac{1}{2}\sqrt{16-4t^4-t^2}$ $\vec{r}(t) = t^2\hat{\imath} + \frac{1}{2}\sqrt{16-4t^4-t^2}\,\hat{\jmath} + t\hat{k}$

choose positive root