Section 6.6 Solving Equations by Factoring
Section 6.7 Modeling and Solving Problems with Quadratic Equations

Math 103 Course Outline Objective:

- Solve a polynomial equation using the graphing method.

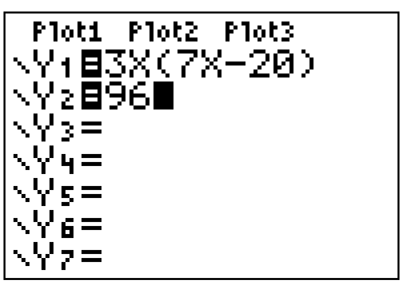
Graphing methods are: “intersection of graphs” and “zero” methods.
(To review the “intersection of graphs” method, see Calculator Guide for Section 4.1.)

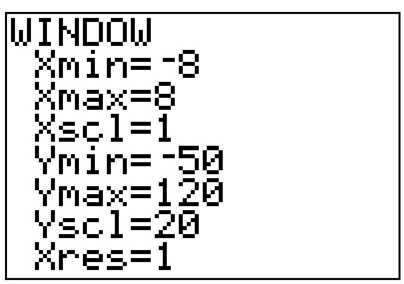
Example 1: Solve $3n(7n-20)=96$ algebraically and check using the intersection of graphs method and the zero method.

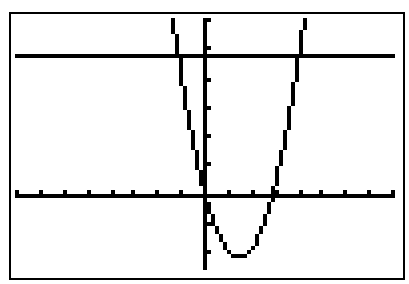
$$\begin{aligned}
&3n(7n-20)=96\\
&21n^2-60n=96\\
&21n^2-60n-96=0\\
&\frac{21n^2-60n-96}{3}=\frac{0}{3}\\
&7n^2-20n-32=0\\
&(7n+8)(n-4)=0\\
&7n+8=0 \quad \text{or} \quad n-4=0\\
&\quad 7n=-8 \qquad\qquad n=4\\
&\quad n=-\frac{8}{7}
\end{aligned}$$

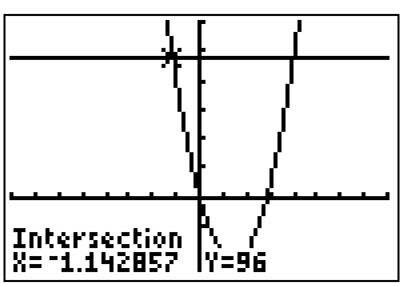
Solution:

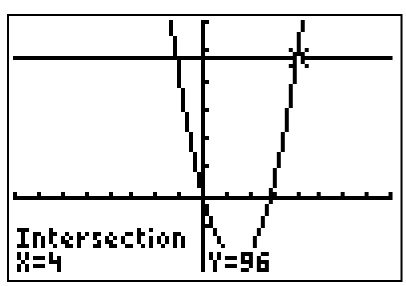
❶ Check using the “intersection of graphs” method:





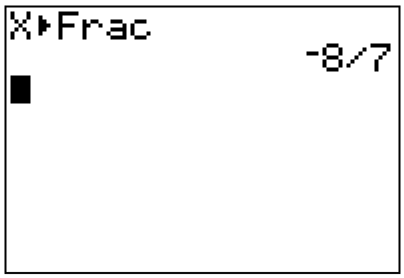






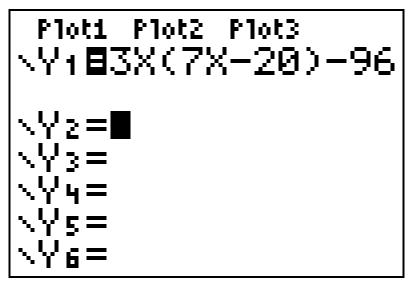
INSTRUCTOR NOTES

- Talk about a suitable viewing rectangle for the equation to be graphed.
- Remind students that the solution is the ***x*-coordinate** of each point of intersection. If the student expresses the solution as (4, 96), for example, that answer is incorrect.
- The calculator stores the last computed value of x. So for Example 1, the x-coordinate of the point of intersection of the two graphs, $x \approx -1.142857$, is stored as x. To change that value to its fractional equivalent, just go to the Home Screen, type in x, from the  key, go to the MATH menu and select the ▸Frac feature, and press ENTER. See the screen below.

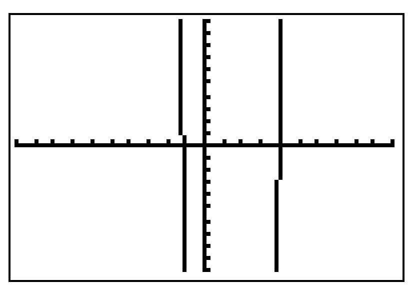


❷ Check using the "zero" method.

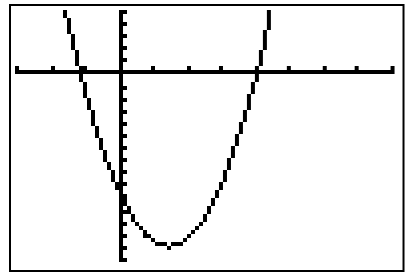
To use the "zero" method, the equation must be in the form *polynomial* = 0. The solutions of the equation are the values of x for which y = 0.



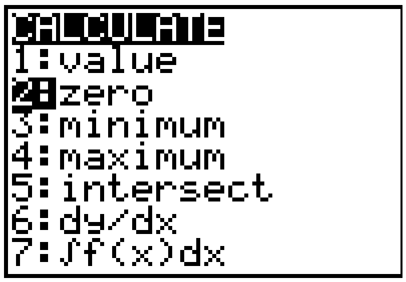
This is the graph in the Standard window. Encourage students to obtain a "better" graph - one in which there's not so much white space, and the vertex can be seen.

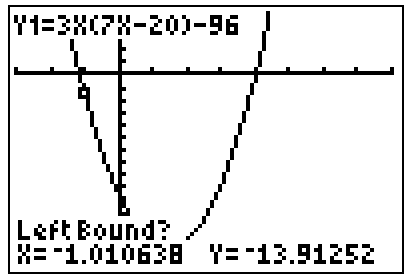


Graph in window [-3, 8, 1] by [-150, 50, 10]:

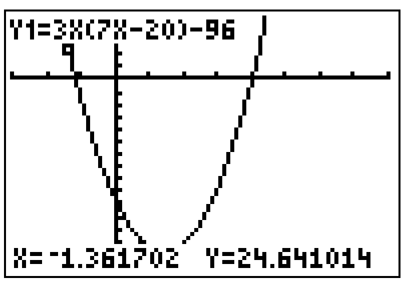


To find the zero feature of the calculator, press 2ND CALC, and you'll see the screen below, on the left. Highlight "zero", and press ENTER. You will see a screen similar to the one on the right, below.

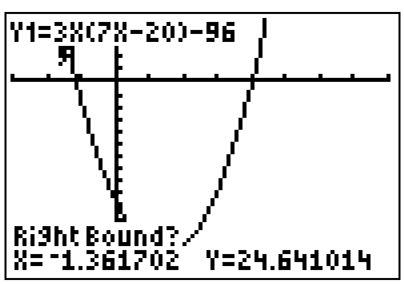




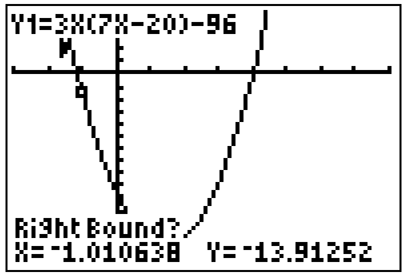
Notice the prompt "Left Bound?". Use your cursor key (left arrow key) to place your cursor slightly to the ***left*** of the x-intercept, as shown in the next screen. Your cursor should be above the *x*-axis.



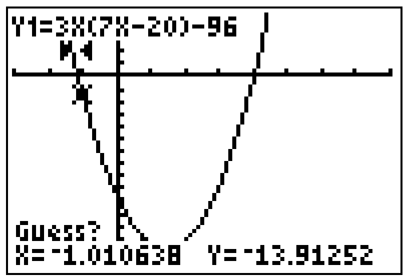
Press ENTER and you'll see the next screen.



You now see a prompt that says "Right Bound?" You must position your cursor to the ***right*** of the *x*-intercept, which in this case will be below the *x*-axis.



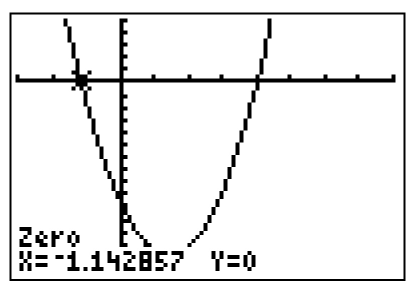
Press ENTER one more time and you see a screen with a prompt that says "Guess?" If you want to move the cursor closer to the *x*-intercept, you may, but be certain to keep the cursor between your left and right bounds. (The bounds are the "barbell" looking icons in the screen that indicate the left and right bounds you chose.)



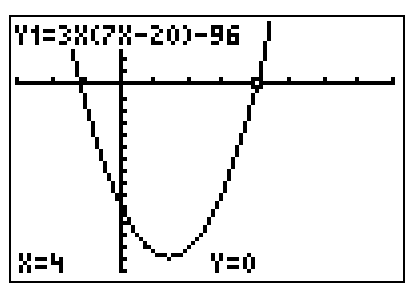
If you do move the cursor closer to the x-intercept, press ENTER after the move. Otherwise, press ENTER immediately after the "Guess?" prompt to allow the program to run.

There it is! The *x*-intercept is approximately -1.142857. You will know that the program is finished when the word Zero appears above the resulting x-intercept.

Solution $x = -\frac{8}{7}$ using "zero" feature:



Because one of the solutions that was obtained algebraically is $x = 4$, students may also just use the VALUE feature of the graph to confirm that when $x = 4$, $y = 0$.



The solutions of the equation $3n(7n-20)=96$ are $-\frac{8}{7}$, 4.

<u>INSTRUCTOR NOTE</u>

Remember to show students how to check solutions to polynomial equations numerically also: home screen substitution and the STO key.

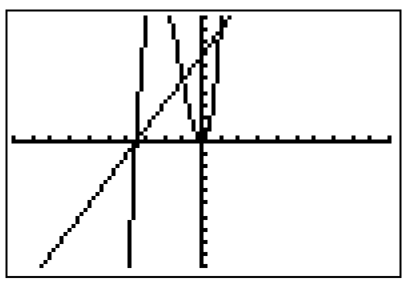
```
3(-8/7)(7(-8/7)-
20)
                96
3(4)(7*4-20)
                96
```

```
-8/7→N:3N(7N-20)
                96
4→N:3N(7N-20)
                96
```

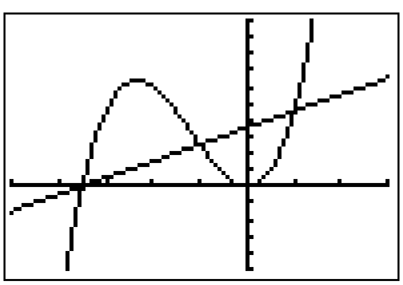
<u>Example 2:</u> Solve the equation graphically: $2x^3+7x^2=2x+7$.

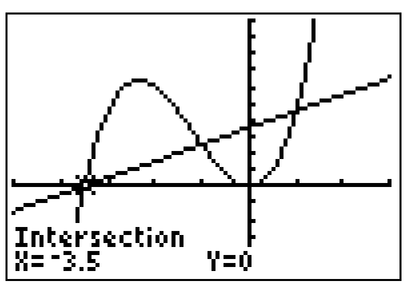
<u>Solution</u>: ❶ Using the "intersection of graphs" method. Students frequently start with the Standard window. But for these graphs, it's best to alter the view from the "Standard" window so that the solutions can more easily be seen.

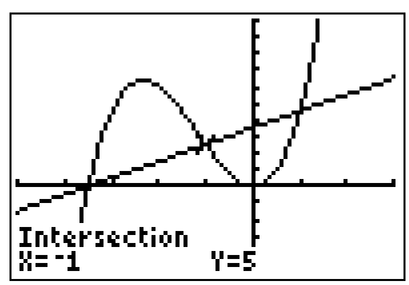
```
 Plot1 Plot2 Plot3
\Y1=2X³+7X²
\Y2=2X+7
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

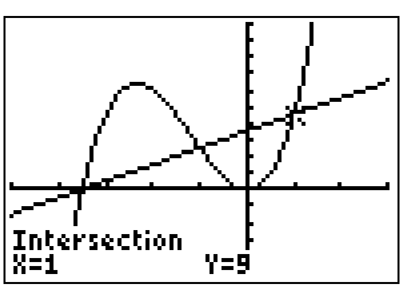


```
WINDOW
 Xmin=-5
 Xmax=3
 Xscl=1
 Ymin=-10
 Ymax=20
 Yscl=2
 Xres=1
```

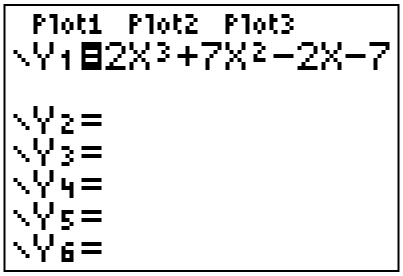


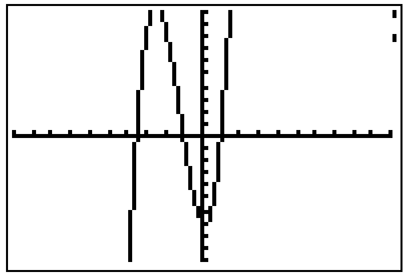


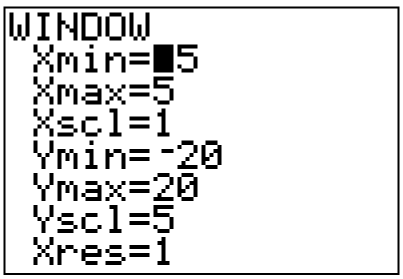


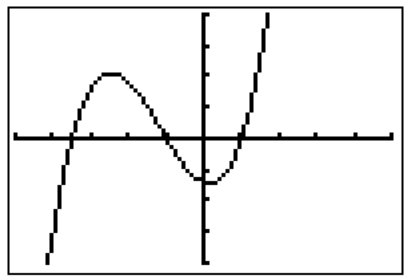


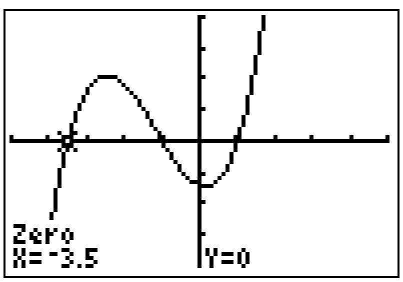
❷ Using the "zero" method. Once again, for these graphs, it's best to alter the view from the "Standard" window so that the solutions can more easily be seen.

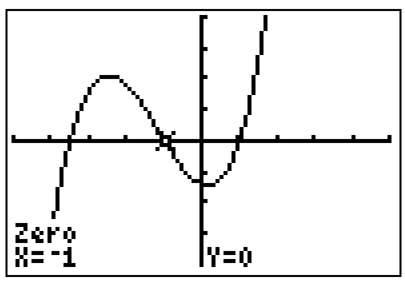


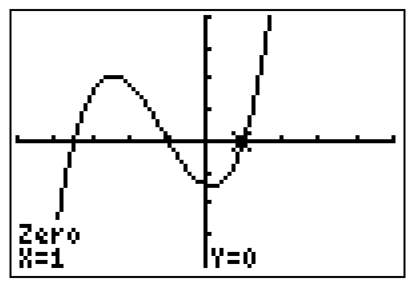












The solutions are the **x-coordinates** of the *x*-intercepts, also called the zeros: the values of *x* for which *y* = 0.

The solutions of the equation $2x^3+7x^2=2x+7$ are -3.5, -1, 1.

INSTRUCTOR NOTES

- ❖ If a solution to an equation is an integer and TblStart and ΔTbl are set properly, then the solution(s) may be identified from the TABLE menu: the solutions *x* = -1 and *x* = 1 for Example 2, for instance.

- ❖ While not a stated course objective, students should know that the solutions of a polynomial equation are the *x*-intercepts of the graph of the related function and that the *x*-intercepts of the graph of a polynomial function are the solutions of the related polynomial equation.

Example 3: An object is thrown upward with an initial velocity of 24 feet per second from the top of the 65-foot high CSCC parking garage. The height above the ground of the object at any time *t* can be described by the equation $h = -16t^2 + 24t + 65$. Find the height of the object at each given time.

a. t = 0 seconds
b. t = 1 seconds
c. t = 1.75 seconds
d. Approximate (to the nearest tenth of a second) how long before the object hits the ground.

Solution: There are multiple ways to find these answers: home screen substitution, using the STO key, selecting the TABLE with Indpnt set on "Ask," or graphing the function and finding the answer from the graph. The "Ask" option is shown below.

a – c.

X	Y1	
0	65	
1	73	
1.75	58	

X=

b. The object will hit the ground when the height = 0. So we solve the equation $0 = -16t^2 + 24t + 65$. From the table above, or the table with Indpnt set on Auto, below, we see that the object hits the ground between 2 and 3 seconds after it was thrown.

X	Y1	
0	65	
1	73	
2	49	
3	-7	
4	-95	
5	-215	
6	-367	

X=0

Because we want the approximation to be to the nearest tenth of a second, show students how to change ΔTbl (the table increment) to 0.1, with TblStart at 2. Then scroll down through the table to find the value of *x* for which Y1 is close to zero.

TABLE SETUP
TblStart=2
ΔTbl=.1
Indpnt: Auto Ask
Depend: Auto Ask

X	Y1	
2.5	25	
2.6	19.24	
2.7	13.16	
2.8	6.76	
2.9	.04	
3	-7	
3.1	-14.36	

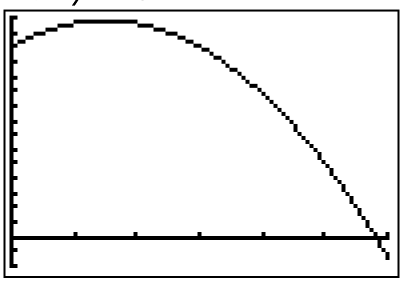
X=3.1

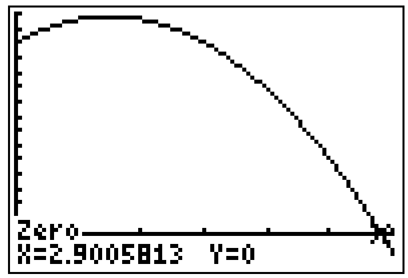
It can be seen that the object hits the ground after approximately 2.9 seconds

INSTRUCTOR NOTE

You may choose to show the "zero" method for answering part (d), the question "Approximate (to the nearest second) how long before the object hits the ground."

If you do so, include a discussion of a suitable viewing rectangle, and the use of the "zero" method to find the *x*-intercept of the function: the value of *x* for which $y = 0$.





Viewing rectangle: [0, 3, 0.5] by [-10, 75, 5]