Name ________________________________________________

Homework #3, Math 151, Fall 2008

Instructions: Record final answers and attach pages with work. All work must be shown in order to receive credit. Exact values should be use unless stated otherwise. Simplify all results.

1. Find the derivative of the functions using the product rule and the quotient rule.
   a. $f(x)=\sqrt{x}\sin x$
   b. $g(s)=\sqrt{s}\ \ 4-s^2$ verify your result by distributing and using the power rule
   c. $h(t)=\dfrac{t}{\sqrt{t}-1}$
   d. $k(x)=\ \ x^2+1\ \ ^2$
   e. $m(x)=x^2\left(\dfrac{2}{x}-\dfrac{1}{x+1}\right)$
   f. $n(t)=\ \ x^2-x\ \ \ x^2+1\ \ \ x^2+x+1$
   g. $p(k)=\dfrac{1}{k}-10\csc k$
   h. $q(n)=2e^n\cos n$
   i. $r(\varphi)=\left(\dfrac{\varphi+1}{\varphi+2}\right)\ 2\varphi-5$

2. Find the equation of the tangent line(s) to the graph of $f(x)=\dfrac{x}{x-1}$ that passes through the point (-1,5).

3. Determine where there exist any values of x in the interval [0,2π) such that the rate of change of f(x)=sec(x) and the rate of change of g(x)=csc(x) are equal. If so, find the point(s).

4. Find the second derivative of the functions.

a. $f(x) = \dfrac{x^2 + 2x - 1}{x}$

b. $h(t) = e^t \sin t$

5. List 4 possible notations for the $2^{nd}$ derivative of a function f.

6. Develop a general rule for $\left[ xf(x) \right]^{(n)}$ where f is a differentiable function of x. Start with n=1 (i.e. xf(x)).

7. Find the derivatives of the functions.

a. $f(x) = -3\sqrt[4]{2 - 9x}$

b. $g(t) = \sqrt{\dfrac{1}{t^2 - 2}}$

c. $k(x) = x(3x - 7)^3$

d. $h(v) = v^2 \tan\left(\dfrac{1}{v}\right)$

e. $m(p) = \ln\left(\dfrac{e^p + 1}{e^p - 1}\right)$

f. $n(x) = x^2 e^{2x} - 2xe^x + 2e^x$

g. $a(t) = \dfrac{-\sqrt{t^2 + 4}}{2t^2} - \dfrac{1}{4}\ln\left(\dfrac{2 + \sqrt{t^2 + 4}}{t}\right)$

h. $y = 5^{x-2}$

i. $b(x) = \log_{10}(2x)$

8. Determine the point(s) at which the graph of $f(x)=\dfrac{x}{\sqrt{2x-1}}$ has a horizontal tangent line.

9. Find the first and second derivatives implicitly. Evaluate both at the indicated point. Use the first derivative to find an equation of the tangent line at the given point.

a. $x^3-y^2=0$ (1,1)

b. $\tan(x+y)=x$ (0,0)

c. $3e^{xy}-x=0$ (3,0)

d. $y^2=\ln x$ (e,1)

10. Find the normal line to the curve (perpendicular to the curve/tangent line) at the given point. $x^2+y^2=9$ at $2,\sqrt{5}$ .

11. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ of

a. $y=(1+x)^{1/x}$

b. $f(x)=\sqrt{(x-1)(x-2)(x-3)}$ compare the process to the non-logarithmic process for finding the derivative (i.e. using the chain rule and product rules). Which do you prefer?

12. Show that the two equations are orthogonal at their intersection points. (i.e. their tangent lines are perpendicular).

$$x^3=3(y-1) \text{ and } x(3y-29)=3$$