A Comparison of Methods for Spatiotemporal Modeling of Ice Sheet Elevation in Greenland

In order to better understand the impacts of climate change, significant effort has been made in recent decades to collect data to improve models of the climate, and in particular, of the cryosphere: that part of the Earth that is covered with ice all year. Data on elevation and ice velocity has been collected *in situ* through physical measurements on the ice sheet. Data has been collected by satellites and by flyovers by plane. Each of these measurements occurs in both time and space. The data is collected at varying time intervals, in part because of different methods of collection, which changed over time, and because some parts of the ice sheet are changing faster than the others necessitated collecting data more frequently at some locations than at others. The data is irregular in that it is sparse in some locations and dense in others, taken at irregular frequencies, making analysis complex.[1] Many methods of time series analysis depend on data collected at regularly spaced intervals. Thus, they are unsuitable for the available data, requiring the development of new methods.

Previous work on the data has included processing the data to produce time series on regular 1-sq.km grids. This analysis has produced roughly 150,000 separate time series.[2] Each of these individual time series can be analyzed independently to model the ice sheet elevation on that region. One method for this analysis is using a spline-based approach (ALPS) to model the series.[3] However, this method of analysis processes time series independently from each other. Moreover, several hyperparameters must be set for this model to obtain the "best" fit.

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[1] Beata Csatho, Integration of ICESat-2 Observations into Ice Sheet Elevation Change Record to Investigate Ice Sheet Processes, 2020, Proposal Number 20-ICESAT2-20-0017, NASA.
[2] Ibid.
[3] Prashant Shekhar et al., "ALPS: A Unified Framework for Modeling Time Series of Land Ice Changes," *IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING* (2020).

While this dataset is already quite large and complex from the perspective of time series analysis, data obtained from ICESat-2 will be both richer and larger by several orders of magnitude.[4] The newest data will have to be integrated with the previously collected data without losing the improved resolution and regularity in both time and space.

The goal for the present research is to advance the modeling of these spatiotemporal series and to generate accurate error estimates. Ultimately, ALPS will be applied to the entire Greenland ice sheet data, and that analysis will be compared to the results of applying a Gaussian process regression model to the same data. Points of comparison between the two methods will include not only predictions, but also their error estimates, forecasts, and overall stability. One concern of the spline-based approach in ALPS is that the error estimates may be too small and may not fully capture the true variability of the measurements and the resulting interpolations. A possible complication of the Gaussian process approach is that the method can be more unstable than splines. That said, the Gaussian process regression model does allow for incorporation of the measurement errors into the model.

The Gaussian process model is a relatively new approach to the analysis of time series of ice sheet dynamics. Hugonnet, et al. applied this approach to ice sheet DEMs (digital elevation models) of one-degree square that retained seasonal signal as well as dynamic ice changes interannually.[5] The data for the present study has already been processed by SERAC to contain only dynamic ice sheet changes.[6] An R package called RobustGaSP, developed by Mengyang

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[4] Thorsten Markus et al., "The Ice, Cloud, and land Elevation Satellite-2 (ICESat-2): Science requirements, concept, and implementation," *Remote Sensing of Environment* 190 (2017), https://doi.org/http://dx.doi.org/10.1016/j.rse.2016.12.029.
[5] Romain Hugonnet et al., "Accelerated global glacier mass loss in the early twenty-first century," *Nature* 592, no. April (2021), https://doi.org/https://doi.org/10.1038/s41586-021-03436-z.
[6] Beata M. Csathoa et al., "Laser altimetry reveals complex pattern of Greenland Ice Sheet dynamics," *PNAS Early Edition* (2014), https://doi.org/www.pnas.org/cgi/doi/10.1073/pnas.1411680112.

Gu will be used to produce the Gaussian process regression models. RobustGaSP allows for the creation parallel partial emulations of both the temporal and spatial components of the data.[7]

The ability to generate an accurate model of ice sheet elevation is crucial to applying the work beyond mere historical analysis. The models can be used to interpolate ice sheet elevation on grids of sizes of other than 1-sq.km. When used in climate models, variable grid sizes, of both coarser and finer resolution, are necessary to interface with other parts of the climate like the ocean and atmospheric models. They can also be used to test climate models for their ability to accurately predict past ice sheet behavior where the accuracy of the error bars will be crucial. If the errors are unrealistically narrow, then otherwise accurate climate models may be judged as inaccurate.

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[7] RobustGaSP.